How the quality of a received EPR pair depends on the distances from an EPR source?

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Abstract

Let an EPR source which generates maximally entangled pairs be located so that it has distances L_1 and L_2 to two users. After taking into account various effects like loss of photons, deficiencies in the source and detectors, an entangled pair traveling through the channel may loose its perfect correlation due to errors in the channel. How the entanglement of the received pair depends on the above distances and the local properties of the channels used for this transmission? What is the best location of the source if we want to achieve the highest fidelity? What is the threshold distance beyond which the entanglement of the pair vanishes and becomes useless for using in teleportation. We discuss these problems for the Pauli channel which simulates the effect of optical fibers and possibly the atmosphere on the polarization-entangled photons.

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1 Introduction

Almost any protocol of quantum information processing requires an entangled pair of particles. Traditionally these pairs have been known as EPR pairs after Einstein, Podolsky and Rosen [1] or Bell states after John Bell [2]. It is usually envisaged that there is an EPR source generating EPR states which distributes such pairs to various users or communicating parties [3, 4, 5, 6]. In recent years a great deal of experimental effort has been devoted to production and transmission of entangled photons [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Although these efforts have initially been fueled by the need to test Bell inequality and its generalizations, in more recent years they are mostly motivated by the perspective of possibility of long distance quantum communication and actual implementation of some quantum information protocols.
This interest is so strong that some researchers are even considering to go beyond Earth-bound laboratories and use satellites as transmitter, receiver or relay terminals [17, 11] for quantum communication.

Over the years it has become possible to distribute entangled photons through increasingly long distances. Some of the distances reported are 360 meters [9], 400 meters [8], 1.45 km [13], and 4 km [7] for polarization entangled photons through optical fibers, 10 km [16] and 50 km [14] for time-bin entangled photons through optical fibers and 7.8 km for polarization entangled photons through free space [12].

In all these experiments various loss effects should be taken into account. The first important loss occurs when one or both photons of the pair are absorbed or scattered in the channel and do not reach the detectors. This type of loss has an exponential dependence on distance with a typical attenuation of a few dB per km depending on the wavelength, (e.g. 3.2 dB/km for optical wavelengths [13], 0.35 dB/km for telecommunication wavelengths [14]). Those truly entangled photons which reach the detectors can be detected by precise coincidence counting in a nanosecond window. The coincidence peak is nearly noise free with a signal to noise ratio exceeding 100 [8]. Detection is usually done by silicon-avalanche photo diodes (SAPD) which have dark counts of a few hundred per second, much lower than 10000-15000 counts in the experiment of Weihs et al in 1998 [8] or 60000 count rates in more recent experiments of the Vienna group [12]. One should of course take into account these dark and accidental coincidence counts and subtract them from all the measured coincidence counts to find the actual number of entangled photons received.

Trying to increase the distance over which entangled photons are distributed is a fairly difficult task. The basic problem is the increased photon loss in the channel which lowers the signal to noise ratio at the detectors. Trying to compensate for the photon loss in the channel, one can raise the rate of photon production in the parametric down conversion process, but this makes the source to deviate more from an ideal source of EPR pairs. In fact the output state of parametric down conversion is always an (ideal) Bell state mixed with other no-photon or multi-photon states and a stronger source raises the probability of these unwanted states. Therefore one has to make a compromise between conflicting factors. A theoretical analysis of the situation is therefore necessary to find the actual limitations on quantum communication tasks [18].

In [18] a detailed study of quantum key distribution has been performed taking into account real experimental situations. In that work the main effects which are taken into account are the deficiency of the parametric down conversion mechanism which produces a Bell state only in a coherent superposition with other un-wanted multi-photon states, deficiencies in the detectors which produce dark counts and false coincidence counts and finally photon loss in the channel which is considered to be exponential in terms of

the length of the channel and is modeled by a beam splitter. In this way it has been shown that security of the BBM92 protocol [19] for quantum key distribution can be attained to distances up to 170 km with the assistance of entanglement swapping. Moreover it has been shown that the loss factors are minimized if the EPR source is situated midway between the two parties involved in this protocol.

Overall it seems that with current technology the earth-bound fibre and free-space quantum communication cannot surpass on the order of 100km [16].

After taking all the above losses into account, we are still faced with the problem that such pairs may loose their perfect correlation or anti-correlation in passing through the channel from the source to the detectors.

Usually polarization drifts or partial rotations of photon polarizations from the source to the fibers are corrected by passing them through compensators and a software which calculates these drifts is used to adjust the phase of the singlet and to compensate the in-crystal birefringence [12, 13, 8]. However this compensation is only done at the beginning of the channel and not the whole length of the channel. Even this, is not always easy. For example in a recent experiment of the Vienna group in which entangled photons are produced and sent to a distance of 7.8 km through air, it is argued that the final states may have been more entangled than indicated by their measured violation of Bell inequality due to the partial rotation of polarization states [12].

Unfortunately a characterization of quantum channels in terms of polarization drift does not exists and one can only infer the error rates produced in such channels by subtracting known error rates from the measured loss of visibility. For example in the experiment of [13] with a distance of 7.8 km between the source and the receiver, after entangled photons have been extracted by precise coincidence counting, the average loss of visibility has been around 8 percent, 2.5 percent of which has been accounted for by imperfection of detectors, 1.2 percent by imperfection of the source and the rest has been attributed to the quantum channel. In another experiment [8] with a distance of 400 m, it has been found that polarization drift has been less than 1 percent.
This is all for fibers, and for free air it is known that the atmosphere is almost non-birefringent.

However if we are going to break the barrier of 100 km and do long distance quantum communication by advancing our technology, certainly the errors in the channel no matter how small they are per length, should be calculated and taken into account. In this article we want to see how the entanglement of an EPR pair distributed by a source, diminishes when the two qubits (e.g. polarization states of photons) travel through distances L_1 and L_2 in a Pauli channel to reach the users. Such a channel has been shown to be suitable for modeling thermally fluctuating birefringence of single mode fibers carrying the polarization states of photons [20].

The main quantities that we calculate are the fidelity of the received EPR pair with the initially maximally entangled pair and also the concurrence of the former which turn out to be functions of $L_1 + L_2$. We will find the threshold for $L_1 + L_2$, beyond which an EPR pair looses all its entanglement, as a function of the error parameters of the channel. We will find that if in the Pauli channel only one bit flip error occurs, then the concurrence of the received pair never vanishes except for infinitely long channels. However if two or three bit-flip errors occur, then there is always a threshold length beyond which entanglement of the

received pair vanishes. From current experimental data we estimate that for the depolarization channel, as a model for optical fibers, this threshold distance is more than 34 kilometers.

The rest of this paper is structured as follows: We first determine in section 2 the dependence of total error parameters of the Pauli channel as a function of its length and the value of its local error parameters. Then in section 3 we send a maximally entangled pair through the channel and calculate the fidelity of the final received pair, with the initial pair.

2 Characterization of the Pauli channel error rates in terms of its length

A quantum channel is specified by the types of error operators it introduces on arbitrary states which are transmitted through the channel. In practice we can characterize a short segment of a channel by suitable quantum measurements in the laboratory. It is then possible to infer the characteristics of an arbitrary length of the channel by combining the quantum operations which pertain to each segment. Suppose a segment of the channel is specified by a quantum operation given by the Kraus decomposition

$$\rho_1 \equiv E(\rho) = \sum_k E_k \rho E_k^{\dagger}. \tag{1}$$

Then a sequence of N segments of such a channel is specified by the quantum operation

$$\rho_N \equiv E^N(\rho) = E(E(\cdots E(\rho))). \tag{2}$$

If we allow for a large enough set of operators, a large enough parameter space, we can say that the set of quantum operations is closed under this concatenation. The problem then reduces to finding the flow of parameters under concatenation. By going to the limit of an infinite number of infinitesimal segments, we can obtain the parameters as functions of the length of the channel (or the time duration of the channel for those cases where our channel only stores data).

An important example of a quantum channel acting on a qubit is the Pauli channel which is specified by the following quantum operation

$$\rho_1 := E(\rho) = p_0\rho + p_1\sigma_x\rho\sigma_x + p_2\sigma_y\rho\sigma_y + p_3\sigma_z\rho\sigma_z, \tag{3}$$

where $p_i,\ i = 0, \cdots, 3$ are respectively the probabilities of no error, bit-flip, bit-phase-flip and phase-flip errors [21, 22] and $p_0 + p_1 + p_2 + p_3 = 1$.

Recently it has been shown that this channel can model the effects of some realistic noise on qubits, like the effect of randomly fluctuating magnetic fields on electron spin or thermally induced birefringence of polarization states of photons traveling through optical fibers [20].

Using the properties of Pauli matrices, it is easily verified that the concatenation of two Pauli channels is again a Pauli channel. Therefore we can write

$$E^N(\rho) = p_0^{(N)}\rho + p_1^{(N)}\sigma_x\rho\sigma_x + p_2^{(N)}\sigma_y\rho\sigma_y + p_3^{(N)}\sigma_z\rho\sigma_z. \tag{4}$$

Using the relation $E^{N+1}(\rho) = E(E^N(\rho))$ and the properties of the Pauli matrices, a recursion relation between the error parameters is obtained. Written in matrix from it reads

$$\begin{pmatrix} p_0^{(N+1)} \\ p_1^{(N+1)} \\ p_2^{(N+1)} \\ p_3^{(N+1)} \end{pmatrix} = \begin{pmatrix} p_0 & p_1 & p_2 & p_3 \\ p_1 & p_0 & p_3 & p_2 \\ p_2 & p_3 & p_0 & p_1 \\ p_3 & p_2 & p_1 & p_0 \end{pmatrix} \begin{pmatrix} p_0^{(N)} \\ p_1^{(N)} \\ p_2^{(N)} \\ p_3^{(N)} \end{pmatrix}. \tag{5}$$

This relation can be solved by diagonalizing the matrix and using the boundary conditions $p_i^{(0)} = \delta_{i,0}, i = 0, 1, 2, 3$. The final result can be written compactly using a matrix notation, i.e

$$\begin{pmatrix} p_0^{(N)} \\ p_1^{(N)} \\ p_2^{(N)} \\ p_3^{(N)} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}, \tag{6}$$

where

$$\begin{aligned} \lambda_1 &= (1 - 2p_2 - 2p_3)^N \\ \lambda_2 &= (1 - 2p_1 - 2p_3)^N \\ \lambda_3 &= (1 - 2p_1 - 2p_2)^N. \end{aligned} \tag{7}$$

This shows how the parameters of a channel made of N consecutive segments are related to the parameters of short segments which are usually easier to characterize in practice. If one defines the parameters $\mu_i := \frac{p_i}{l}$ as error per length for very short segments, then the parameters for a channel of length L take the following form

$$\lambda_1 = e^{-2(\mu_2+\mu_3)L}, \quad \lambda_2 = e^{-2(\mu_1+\mu_3)L}, \quad \lambda_3 = e^{-2(\mu_1+\mu_2)L}. \tag{8}$$

Inserting the above values of λ_i in (6) gives the error parameters of the channel in terms of its length.

$$\begin{pmatrix} p_0(L) \\ p_1(L) \\ p_2(L) \\ p_3(L) \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ e^{-2(\mu_2+\mu_3)L} \\ e^{-2(\mu_1+\mu_3)L} \\ e^{-2(\mu_1+\mu_2)L} \end{pmatrix}. \tag{9}$$

As a special case we will have the following result for flip channels, where $\rho_{flip}(L)$ denotes the density matrix at the output of a channel of length L and the index i takes the values $1, 2$ and 3 for the bit-flip, the bit-phase flip and the phase-flip channel respectively:

$$\rho_{flip}(L) = \frac{1}{2}(1 + e^{-2\mu_i L})\rho + \frac{1}{2}(1 - e^{-2\mu_i L})\sigma_i \rho \sigma_i. \tag{10}$$

We can see that for very long channels where $L \to \infty$, and we will have $\rho_{flip}(\infty) = \frac{1}{2}(\rho + \sigma_i \rho \sigma_i)$. Thus the probability of flipping tends to $\frac{1}{2}$ as expected for the worst case of a flip-channel.
Another special case is the depolarizing channel which is defined by the quantum operation [21, 22]

$$E(\rho) = p\frac{I}{2} + (1 - p)\rho, \tag{11}$$

or equivalently by equation (3) with parameters $p_0 = 1 - \frac{3p}{4}, p_1 = p_2 = p_3 = \frac{p}{4}$. Using (9) with $\mu := \mu_1 = \mu_2 = \mu_3$ we find $P_{0,\text{depol}}(L) = \frac{1}{4}(1 + 3e^{-4\mu L})$ and $P_{i,\text{depol}}(L) = \frac{1}{4}(1 - e^{-4\mu L}), \;\; i = 1, 2, 3$. Using the well known identity $\sum_i \sigma_i \rho \sigma_i = 2I - \rho$, we can rewrite this as

$$\rho_{depol}(L) = e^{-4\mu L}\rho + \frac{1}{2}(1 - e^{-4\mu L})I. \tag{12}$$

As $L \to \infty$ the probability of error tends to 1 and we will have $\rho_{depol}(\infty) = \frac{I}{2}$ which is a completely mixed state carrying no information of the original state.

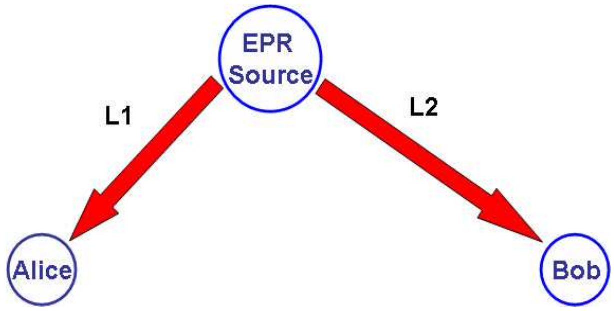

Figure 1: "(Color online)" Transmission of EPR pairs from a source to Alice and Bob trough channels with different lengths L_1, L_2.

3 Transmission of EPR pairs through Pauli channels

We now consider two users with distances L_1 and L_2 to an EPR source (figure(I)). The source prepares a maximally entangled pair and sends each qubit of the pair to a user. We want to calculate the efficiency of this process, measured by the concurrence of the received pair as a function of distances L_1 and L_2 and the error parameters of the channel, which are assumed to be of the same type for both routes.

Suppose that the source sends a maximally entangled state $\rho = |\psi^+\rangle\langle\psi^+|$ into the channel where $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0,0\rangle + |1,1\rangle)$. The parameters of the two channels are denoted respectively by $r_i := p_i(L_1)$ and $s_i = p_i(L_2)$ derived from equation (9). After transmitting the state through the Pauli channel, this state is transformed to

$$\rho' = r_0 s_0 \rho + r_0 \sum_{k=1}^{3} s_k (I\otimes\sigma_k)\rho(I\otimes\sigma_k) + s_0 \sum_{k=1}^{3} r_k(\sigma_k\otimes I)\rho(\sigma_k\otimes I) + \sum_{k,l=1}^{3} r_k s_l (\sigma_k\otimes\sigma_l)\rho(\sigma_k\otimes\sigma_l). \quad (13)$$

Evaluation of the right hand side is facilitated by noting that $\rho = |\psi^+\rangle\langle\psi^+|$ can be written as $\rho = \frac{1}{2}(S - \sigma_y\otimes\sigma_y)$, where S is the swap operator $S|\alpha,\beta\rangle = |\beta,\alpha\rangle$. One then uses the following easily verified identities

$$\sigma_k\sigma_y\sigma_k = (-1)^k\sigma_y, \quad S(A\otimes B) = (B\otimes A)S, \quad (14)$$

and $\sum_{k=1}^{3}\sigma_k\otimes\sigma_k = 2S - I$.
A rather lengthy but straightforward calculation will determine the output density matrix. It is given by

$$\rho' = a|\psi^+\rangle\langle\psi^+| + b|\psi^-\rangle\langle\psi^-| + c|\phi^+\rangle\langle\phi^+| + d|\phi^-\rangle\langle\phi^-|, \quad (15)$$

where

$$\begin{aligned} |\psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|0,0\rangle \pm |1,1\rangle), \\ |\phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|0,1\rangle \pm |1,0\rangle), \end{aligned} \quad (16)$$

are the Bell states and

$$\begin{aligned} a &= r_0s_0 + r_1s_1 + r_2s_2 + r_3s_3 \\ b &= r_0s_3 + r_1s_2 + r_2s_1 + r_3s_0 \\ c &= r_0s_1 + r_1s_0 + r_2s_3 + r_3s_2 \end{aligned}$$

$$d = r_0s_2 + r_1s_3 + r_2s_0 + r_3s_1. \tag{17}$$

These parameters are in fact the fidelities of the output state with the Bell states $|\psi^+\rangle,\ |\psi^-\rangle,\ |\phi^+\rangle$ and $|\phi^-\rangle$ respectively.
Using the relations (9), (17) take the following form

$$\begin{aligned}
a &= \frac{1}{4}\{1 + e^{-2(\mu_1+\mu_2)L} + e^{-2(\mu_1+\mu_3)L} + e^{-2(\mu_2+\mu_3)L}\} \\
b &= \frac{1}{4}\{1 + e^{-2(\mu_1+\mu_2)L} - e^{-2(\mu_1+\mu_3)L} - e^{-2(\mu_2+\mu_3)L}\} \\
c &= \frac{1}{4}\{1 - e^{-2(\mu_1+\mu_2)L} - e^{-2(\mu_1+\mu_3)L} + e^{-2(\mu_2+\mu_3)L}\} \\
d &= \frac{1}{4}\{1 - e^{-2(\mu_1+\mu_2)L} + e^{-2(\mu_1+\mu_3)L} - e^{-2(\mu_2+\mu_3)L}\},
\end{aligned} \tag{18}$$

where $L := L_1 + L_2$. It is obvious that $a > b, c, d$ so the fidelity of transmitted pair with $|\psi^+\rangle$ is greater than the other Bell states.
We are interested in the concurrence of the state ρ' [23], since it is a measure of the degree of mutual entanglement of the received pair of qubits and hence a measure of the success of any quantum protocol like teleportation which may use this pair.
Using the result of [23], the concurrence $C(\rho')$ can be calculated in a straightforward way. It is given by

$$C(\rho') = max(0, 2\alpha_{max} - 1), \tag{19}$$

where $\alpha_{max} := \max(a, b, c, d)$. Note that a nonzero concurrence implies that $\alpha_{max} > \frac{1}{2}$ which in turn implies that the fidelity of the state ρ' with one of the Bell states is greater than $\frac{1}{2}$. Under these conditions one can use the state ρ' to achieve a fidelity of teleportation exceeding the one obtained in the best classical protocols [24, 25]. Using (18) and (19) we obtain

$$C_{\text{Pauli}}(\rho') = max(0, \frac{1}{2}\{e^{-2(\mu_1+\mu_2)L} + e^{-2(\mu_1+\mu_3)L} + e^{-2(\mu_2+\mu_3)L} - 1\}) \tag{20}$$

Thus the fidelity and the concurrence depend only on the sum of the distances and not on the individual distances. Specially if the EPR source is collinear with the users and situated between them, then the location of the source is immaterial to the efficiency.

3.1 Single Bit-flip channels

As a special case we study the bit-flip channel for which $\mu_2 = \mu_3 = 0$. Equations (15) and (18) show that

$$a = \frac{1}{2}(1 + e^{-2\mu L}), \quad b = 0, \quad c = \frac{1}{2}(1 - e^{-2\mu L}), \quad d = 0. \tag{21}$$

Using (15, 17) and (20) we find the fidelity of the output state with the initial Bell state $|\psi^+\rangle$ and its concurrence to be

$$\begin{aligned}
\langle\psi^+|\rho'|\psi^+\rangle &= \frac{1}{2}(1 + 2^{-2\mu_L}) \\
C_{\text{bit-flip}}(\rho') &= e^{-2\mu_1 L}.
\end{aligned} \tag{22}$$

This shows that for the bit flip channel, no matter how long the channel is, the received state can always be used for teleporation or some other quantum protocol, possibly after some distillation to increase the

efficiency. This result is also valid for the other single flip-channels. Note that in this case the output density matrix is a mixture of only two Bell states and this mixture is 50-50 only when the length of the channel goes to infinity. This is in agreement with a result of Horodecki's [26] which state that a mixture of two Bell states is always entangled , except when the mixture is 50-50.

3.2 Double bit-flip channels

If the channel allows for more than one type of flip error, then there is always a threshold distance beyond which the received state is useless. To see this consider the case where $\mu_3 = 0$ and $\mu_1 = \mu_2 = \mu$. In this case we find from (20) that

$$a = \frac{1}{4}(1 + e^{-2\mu L})^2, \quad b = \frac{1}{4}(1 - e^{-2\mu L})^2, \quad c = d = \frac{1}{4}(1 - e^{-4\mu L}). \tag{23}$$

From this we obtain

$$\begin{aligned} \langle\psi^+|\rho'|\psi^+\rangle &= \frac{1}{4}(1 + e^{-2\mu L})^2 \\ C_{\text{double-flip}}(\rho') &= \frac{1}{2} max(0, e^{-4\mu L} + 2e^{-2\mu L} - 1), \end{aligned} \tag{24}$$

which implies that beyond a threshold length

$$L^{\text{th}}_{double-flip} := \frac{1}{2\mu} \ln\left(\frac{1}{\sqrt{2} - 1}\right) \tag{25}$$

the concurrence vanishes.

3.3 Depolarization Channel

For the depolarizing channel where $\mu_1 = \mu_2 = \mu_3 = \mu$ we find from (20) that

$$a = \frac{1}{4}(1 + 3e^{-4\mu L}), \quad b = c = d = \frac{1}{4}(1 - e^{-4\mu L}). \tag{26}$$

From this we obtain

$$\begin{aligned} \langle\psi^+|\rho'|\psi^+\rangle &= \frac{1}{4}(1 + 3e^{-4\mu L}) \\ C_{\text{depol}}(\rho') &= \frac{1}{2} max(0, 3e^{-4\mu L} - 1), \end{aligned} \tag{27}$$

which implies that beyond a threshold length

$$L^{\text{th}}_{depol} := \frac{\ln 3}{4\mu} \tag{28}$$

the concurrence vanishes, rendering the transmission useless. In figure(II), $C_{depol}(\rho')$ is plotted in terms of L for two different values of μ.

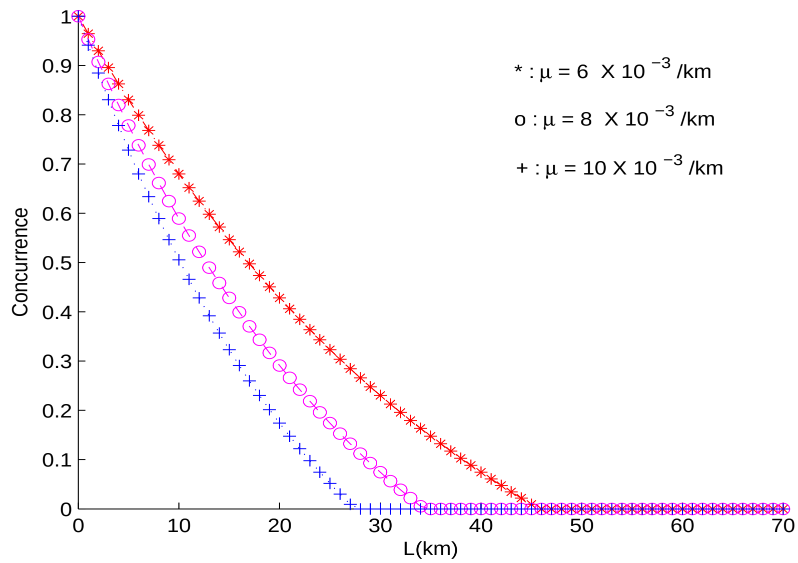

Figure 2: "(Color online)" Concurrence versus total length $L = L_1 + L_2$ in depolarizing channel for two different values of μ .

4 Relation with experiments

Assuming that a depolarization channel can model the polarization drift of photons in an optical fiber, we can make a prediction as to how long two photons can travel through this media before their correlations are totally lost. In the experiment of Weihs et al [8], it is reported that the polarization drift has been less than 1 percent after the photons have traveled a distance of 400 meters. Inserting this value in equation (27) one deduces a value of $\mu \approx 8 \times 10^{-3}/km$. In a more recent experiment [12], from an average QBER (qubit error rate) of 8 percent, 2.5 percent is accounted for by imperfection of detectors, 1.2 percent by imperfection of photon sources and the rest 4.3 percent by the errors in the quantum channel. Inserting this in equation (27) for $L = 1.45km$ [13], gives a value of $\mu \approx 10 \times 10^{-3}/km$. If we take a tentative value of $\mu \sim 8 \times 10^{-3}/km$ for transmission of polarization-entangled photons through optical fibers, then from equation (28) we find a threshold distance of about 34km, beyond which the concurrence of the final EPR pair drops below zero.
This value is much longer than the current distances over which polarization-entangled photons have been distributed along fibers, and lower than the distance limit of about 60 km that time-bin entangled photons can be distributed [14]. However it is less than the limit of 170 km found in [18] which has been found mainly on the basis of a compromise between increasing the photon production (to overcome absorbtion) and increasing the efficiency of the EPR source (the parametric down conversion process). At present experimental data can not determine which of the above bounds is more stringent. Moreover if we note that part of the errors in the cited experiments above is due to polarization misalignments [13], then the estimated value of μ will become smaller leading to threshold distances longer than 34 kilometers.

5 Discussion

Given a transmission line of energy, i.e. electric power, optical signals, it is always possible to determine the loss of energy in terms of the local properties of the line and the distance from the generator. We have asked this same question for a class of quantum channel, namely the Pauli channel and have derived expressions of error parameters in terms of the local error densities (probability of error per length) and the length of the channel. We have then considered an EPR source, which is to send a maximally entangled pair to two users which have distances L_1 and L_2 to the source. We have calculated the entanglement of the received pair and its fidelity with the original pair as a function of these distances. For the Pauli channel the concurrence of the final received pair depends only on $L_1 + L_2$, the sum of the two distances. In the special cases where the source is collinear with the users and is located between the users, our results show that for the Pauli channel, the efficiency is independent of the location of the source, although to minimize other losses the best location of the source turns out to be midway between the source and the receivers [18]. By using some of the current experimental data we have determined a threshold distance beyond which the correlation of the initial EPR pair drops to zero. This distance certainly is greater than 34 kilometers. In order to find more precise values of threshold distances one should have a characterization of local error parameters of optical fibers.

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